Overcoming the Technical Nature in Learning Translation: *Cabri Geometry*, a Tool to Sustain Conceptual Comprehension?

Annie Corriveau

Faculty of Education
University of Sherbrooke
annie.z.corriveau@usherbrooke.ca

Abstract

This article presents the results of a didactic experimentation conducted with two elementary pupils (11-12 years of age) in a Quebec school in the of learning geometric translation with a mathematical software, called *Cabri Geometry*. The teaching-learning sequence experimented with Barth's (2001) process of conceptualization. The working strategies of pupils were analyzed in relationship to this process. Their comprehension of the concept of translation was considered before and after the teaching-learning sequence using the Herscovics and Bergeron (1988) model of comprehension of conceptual schemata. Results indicated that by following the teaching-learning sequence, the logical mathematical comprehension of the concept of translation evolved in the pupils, while the logical- physical comprehension of this concept remained inaccurate. Moreover, the results revealed different utilization procedures of *Cabri Geometry* software. These utilization procedures likely affected the process of conceptualization and, consequently, the comprehension of pupils.

Introduction

Most educational systems in teaching and learning encourage the use of information and communication technologies [ICT] (OCDE, 2001). This is the case in Quebec, Canada, which makes it a competency, namely 'to exploit information and communication technologies', in new preschool and elementary education programmes (Government of Quebec, 2001). In a section specific to the field of mathematics, it is mentioned that the use of ICT is mandatory. However, the choice of technological tools and the manner in which to use them remains at the discretion of the teacher.

In this article, the results of a didactic experimentation are presented, deriving from a framework previously designed from graduate research (Corriveau, 2007). A dynamic geometry software program called *Cabri Geometry* was deployed for the learning of the concept of translation by two elementary pupils (11-12 years). Two aspects were analyzed: the process of conceptualization during the teaching-learning sequence, and the comprehension of pupils following it. The second aspect is the concern of this paper.

The Problem

The field of geometry was studied as this field of mathematics is often left aside from others, such as arithmetic, according to scientific literature. Moreover, problems with teaching and learning are associated with geometry. In fact, pupils generally have difficulty reasoning geometrical figures; that is, they are challenged by the sorting out of geometrical properties to link these with geometrical objects (Laborde & Capponi, 1994). In addition, pupils frequently develop a limited comprehension of geometrical concepts since teachers usually present them with a restricted set of examples (Battista, 2001). In our opinion, this only serves to exacerbate the links between learning difficulties and the manner with which geometrical concepts are taught. In this respect, Perrin-Glorian (2003) stressed that the teaching of geometry resides almost solely in the technical utilization of geometric instruments.

When confronted with these problems, it seems necessary to conceive of a teaching- learning device susceptible to help pupils participate actively in their process of conceptualization so that, in the long run, they will improve their comprehension of geometric concepts. Research in the teaching of mathematics suggests that the utilization of dynamic geometry software (DGS) for the learning of geometrical notions is important, because it enables pupils to understand geometrical concepts (Assude & Gelis, 2002; Laborde, 2000; Laborde & Capponi, 1994). The notion of ‘dynamic geometry’ refers to two phenomena: the possibility of modifying an existing figure by using the drag mode, and the possibility of maintaining properties of a geometrical construction in the moving of basic objects, which served in its development (Laborde, 2000).

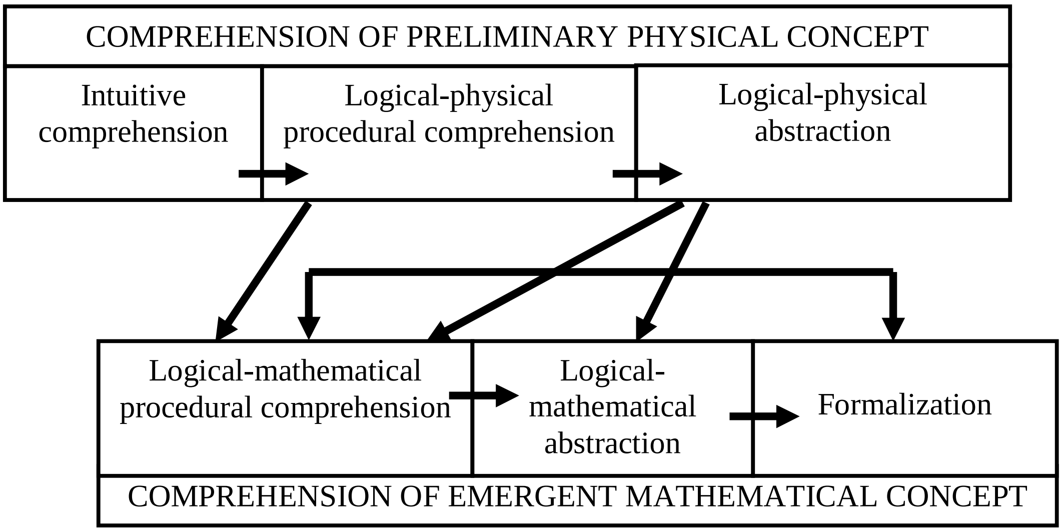
Starting with this information, our basic postulate is that DGS may be used to learn a new geometric concept, because the drag mode enables the modification of a geometric figure. This mode would favour the exploration of a large number of examples for a given concept. In addition, given the dynamic nature of the DGS, the concept of translation represents an interesting notion to work with this kind of tool since it helps visualize the movements made, which assists in discovering the properties relative to an initial figure and those of the image of this figure. By relying on this postulate, our objective is to answer the following question: how does the use of dynamic geometrical software sustain the process of conceptualization as well as favour a better comprehension of the concept of translation in elementary school pupils?

Conceptual Framework

The conceptual framework for this research deals with the process of conceptualization as well as the comprehension of mathematical conceptual schemata. The study of the conceptualization process draws from the works of Barth (2001), who proposed a teaching-learning process of concepts relying essentially on the presentation of examples and counter-examples; they were chosen with care with consideration of attributes to be discovered. According to Barth (2001) “a concept claims in its definition the choice of a negative case, and in helping explore the contrast, we help the learner to structure the elements that he meets" (p. 69). The usage of counter-examples therefore helps the pupil distinguish the essential attributes of a concept from those that are not essential. In order to do this, he or she must use mental strategies such as perception, comparison, and inference.

To illustrate pupil comprehension, the model by Herscovics and Bergeron's (1988) has been chosen. This model attempts to represent different dimensions of comprehension of conceptual schemata in mathematics. This model, illustrated by Figure 1, is divided into two stages of comprehension.

Figure 1: Model of Comprehension of Conceptual Schemata in Mathematics proposed by Herscovics & Bergeron (1988)



Note: This figure represents the components of the model of comprehension chosen as well as the non-linear nature of the process of comprehension.

This model illustrates the complexity of the process of construction of a concept that is non-linear. In fact, a pupil may pass from a logical-physical comprehension procedure to a logical-mathematical comprehension procedure without having abstracted the elements linked to the physical concept, that is, the pre-concept. In other words, a pupil may attain the second stage without necessarily having acquired the components of the first. Nevertheless, there exists a form of hierarchy between the first two components of the first stage, because the acquisition of logical-physical procedural comprehension requires an intuitive comprehension of the concept.

The first stage deals with comprehension being linked to the pre-concept, and refers to a reflection on physical objects. The steps involved include:

1. *Intuitive comprehension* concerns the global perception of the concept. At this level, the pupil possesses incomplete representations of the geometric translation. He or she may recognize situations representing movements of parallel slides as well as those which are not.

2. *Logical-physical procedural comprehension* pertains to the acquisition of procedures that the pupil uses with physical objects while associating these procedures with intuitive knowledge. In addition to recognizing the movement associated with the translation, the pupil is capable of doing a translation himself or herself by completing rudimentary procedures. For example, he or she would be able to move an object from one point to another by making a parallel slide without raising, rolling, or turning it.
3. *Logical-physical abstraction* has to do with the construction of invariants relative to the spatiotemporal transformation as well as generalizations of physical objects. At this level, a pupil recognizes that the object does not change its dimension or orientation during its translation.

The second stage concerns the comprehension of emerging mathematical concepts, and therefore deals with mathematical entities, thus, abstractions. The steps involved include:

1. *Logical-mathematical procedural comprehension* refers to the acquisition of explicit procedures that the pupil can link with preliminary physical concepts and use in an appropriate fashion. The pupil has the capacity to perform a translation of an object or a figure in relation to a direction, sense and given length by using a procedure and appropriate tools.
2. *Logical-mathematical abstraction* deals with the construction of logical-mathematical invariants. The pupil can link them to logical-physical invariants. The pupil knows that the object or the figure remains constant during movement; in other words, their measures of length and angle, parallelism and orientation are conserved.
3. *Formalization* refers to the symbolization of notions for which a degree of procedural comprehension or abstraction already exists. A pupil masters the formalization of a translation through identification and by specifying the starting point or arrival point, as well as the distance, sense, and direction of the vector.

This article intends to present the evolution in appropriation by pupils of the concept of geometric translation using the model of comprehension given by Herscovics and Bergeron (1988), which results from the sequence of teaching-learning conceived according to the process of Barth (2001). This process includes the utilization of a DGS.

Methodology

Given that geometric translation is part of the third cycle of elementary programming in Quebec, two sixth grade pupils were chosen to participate in this research, and shall be called François and Jean. It is important to mention that the concept of translation had not been taught to these pupils previously; however, the pupils stated they did comprehend the meaning of reflection. In order to choose the dynamic geometry software, we proceeded with a criterion-based evaluation of four DGS, specifically, *Geometer's Sketchpad*, *Geogebra*, *Geonext* and *Cabri Geometry*. This form of evaluation enabled us to establish that *Cabri Geometry* seemed most appropriate for elementary pupils to learn geometric transformations (Corriveau and Morelli, 2005). In order to answer our research question, we opted for data collection methods allowing us: first, to establish the evolution of comprehension by pupils of the concept of translation, and second, to describe the process of conceptualization during the sequence. These data were collected by the researcher through means of interviews and participant observation, which was filmed.

The interview served to gather information on the comprehension of pupils regarding the concept of translation. We conducted a pre- and a post-interview, identical from every point of view, over a two-week period. The nature of the interview conducted was similar to the mini-interview designed by Nantais (1992) to inquire into the comprehension of pupils of a conceptual schema. This type of interview relied on a task-dialogue combination between the interviewer and the pupil focusing on a specific assignment on a given mathematical notion. In order to observe the activity of the pupil and question, this method focused on the discovery of his or her cognitive process and further promoted the validation of participant's words and actions. Interview questions were elaborated from a conceptual analysis of the notion of translation. These questions were specifically chosen to target the elements belonging to each of the components of Herscovics and Bergeron's (1988) model, as illustrated in the following example, which is modelled on the second stage (refer to Figure 1). Such questions were: "In the first figure, I have a side (AC) which measures 3 centimetres. Could you tell me what you would do to find the measurement of (A'C') [corresponding side on the image figure]?"

One of the tasks proposed to the pupils was to perform a translation on a sheet of paper with geometrical instruments. As for the first stage, one of the tasks of the pupil was to perform a translation of a cylinder and one of an octagonal prism, whereas one of its requirements, during the course of the activity, was to name, from the different illustrations, one or more choices that represented a parallel slide. Even though the questions were identical for both pupils, the interview took place separately to identify the reasoning associated with the choices made by each pupil.

Participant observation was employed to describe the process of conceptualization of pupils during the learning of the concept of translation with *Cabri Geometry*. With this method, our objective was to understand the procedure used by both pupils during the course of the teaching-learning sequence. To do this, a video camera was used to film the actions of pupils during the activity and during their interactions with the researcher. Also, in order to gather information impossible to obtain with through video taping, namely mental representations and decisions related to the use of the software, we questioned the two pupils during the course of their activity. By doing so, we aimed to identify their choices and motivations for their actions, which were still fresh in their memory.

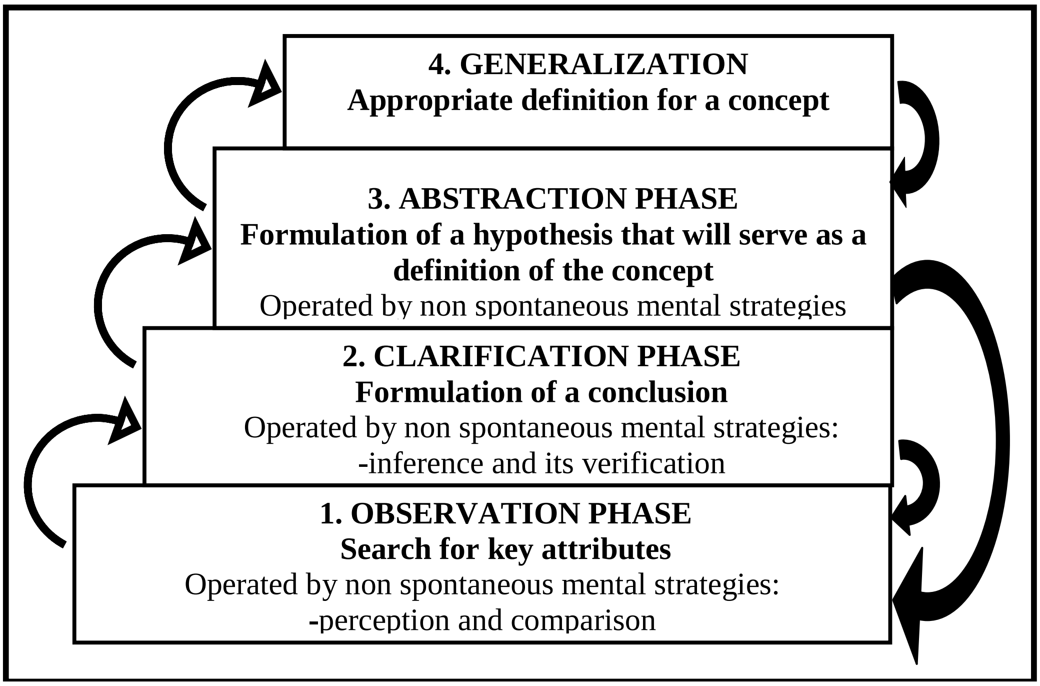
The experimentation took place in three intervals during a total period of three weeks. The first week was dedicated to the passing of the pre-interview. Each of these interviews took approximately 15 minutes. The interview data identified the pupils' level of comprehension regarding translation, before the teaching-learning sequence. The second week was spent experimenting with the teaching-learning sequence. This sequence was divided into three activities of approximately 20 minutes of familiarization with the software, and two specific periods of about 30 minutes for the learning of geometric translation. The third, and final, week was for post-interviews to examine the evolution of comprehension of pupils following the teaching-learning sequence.

Interview and observation data were transcribed verbatim. Their content was then analyzed by means of content analysis (Bardin, 2003). Using a categorial mixed system, in which some categories stem from a conceptual framework and others from the data, such as the static or dynamic aspect of mental strategies of observation and comparison. For the purposes of explanation, the analysis of pupils' comprehension was done by means of a categorical system elaborated from the model of comprehension retained. Hence, for each verbatim transcription, which contained the words and the description of actions of pupils, the units of meaning were coded, on the one hand, according to the component they referred to, and on the other hand, according to the accuracy, from a mathematical point of view, of what the pupil said or did.

Conception of the Teaching-Learning Sequence

The teaching-learning sequence, based on the process proposed by Barth (2001), is an inductive process. It demands that pupils construct the sense of the concept of translation, starting with the exploration of several examples and counter-examples. However, the teaching-learning procedure differs from Barth's work, in that the sequence unfolds individually rather than in a group. To illustrate the translation, examples were chosen that varied in type of figure and information given by the vector (sense, direction, distance). For each example, pupils could manipulate, with the assistance of the drag mode, the initial figure and the vector to verify the invariants and to bring out the essential attributes of translation, notably the role of the vector. In the case of counter-examples, we chose to illustrate other geometric transformations (rotations, reflections) that shared attributes essential to the translation (isometric character), but distinguishable from the type of movement allowed (returning, turning). Also, among the counter-examples, there were cases of geometric transformations and some with no geometric transformation whatsoever. Figure 2 illustrates the mental strategies of each phase of the sequence.

Figure 2: Acquisition Phases of a Concept According to the Approach Proposed by Barth (2001)



Note: This figure is used to illustrate the different phases of the teaching-learning sequence.

In the first phase, pupils must resort to two mental strategies of perception and comparison. In doing so, they had to observe and compare examples and counter-examples while manipulating the information within the examples to bring out differences and similarities (Figure 2). During the second phase, pupils had to make inferences from what they believed to be the list of essential attributes that enable translation. They then had to verify their inference until they obtained a just conclusion. During the third and final phase, once the inference is judged to be adequate, pupils must assure that the generalization of their list of attributes applied to new examples of translation. To do this, they had to establish the hypothesis that their inference may serve to define the translation and, subsequently, verify this hypothesis by resorting to new examples of translation. Finally, to verify the acquisition of the concept, pupils were expected to find a way to perform a translation with *Cabri Geometry*.

Results

The results of the study were presented in a way that highlighted the similarities and distinctions between the evolution in comprehension of pupils relative to the model of comprehension of conceptual schemata as conceived by Herscovics and Bergeron (1988). However, in an attempt to explain the comprehension of the pupils and application of the tool, the components of the process of conceptualization were highlighted, such as mental strategies, as well as procedures for utilization of the tool.

Evolution of Comprehension

The analysis of the pre-interview results indicated that at the outset the two pupils had an inaccurate comprehension of geometric translation, both on the logical-physical plane and the logical-mathematical plane. For example, in the case of the latter, the following excerpt suggests that the term 'translation' had no meaning for François. He stated, "I do not really understand what you mean by translation." Moreover, when he tried to do a translation on paper, he placed the figure randomly rather then considering the information given by the vector. As for Jean, he also questioned what he had to do when we asked him to do a translation. He asked, "Do I have to reproduce it?" On the logical-physical plane, when we asked François to make a cylinder slide in a parallel way, he made it roll when the cylinder was on its curved face, and turned it when it was on its plane face. As for Jean, he no longer understood the meaning of parallel slide. He stated that an object that turns, such as a big wheel, creates a slide. According to him, an object must slide in order to turn.

The interview data showed that the teaching-learning sequence allowed François and Jean to learn and modify their comprehension within the given sequence. However, it is interesting to note that this evolution concentrated in mostly the second stage, that of logical-mathematical comprehension. In fact, logical-physical comprehension of the concept of translation of the two pupils had remained inaccurate, notably the elements linked to the pre-concept. Conversely, they attained a solid logical-mathematical comprehension that differed from each other. This constituted another noteworthy aspect of the study. To better understand this distinction, we attempted to link this observation with mental strategies and procedures used by pupils during the activity. Before establishing links between comprehension and the process of conceptualization, we looked at the specific learning of the two pupils as well as the main strategies and procedures they employed when using the software.

Learning Acquired by François

Analysis of the post-interview results, in relation to the model of comprehension of conceptual schemata as defined by Herscovics and Bergeron (1988), indicated that at the level of formalization François acquired a good comprehension of the role of the vector in translation. He was capable of maintaining that the vector indicated the length and direction of the translation, and he was capable of taking this into consideration when the time came to create a translation on paper using geometrical instruments. On the logical-mathematical plane of abstraction, François also developed a solid understanding of abstraction relative to translation, such as the conservation of measurement of length and of angle, as well as the conservation of the orientation of the figure during movement. It is important to specify that at the beginning he verified, using his ruler, the measurement of one side of the transformed figure. This assured him that the measurement was identical to the corresponding side of the initial figure. After this verification, François seemed convinced, however, that the lengths of the side of the transformed figure were identical to those of the initial figure. On the plane of logical-mathematical procedural comprehension, François was capable of doing a translation on paper by regarding the length of the vector. Moreover, it is surprising that he used, on occasion, a procedure that enabled him to conserve the direction of the vector. In fact, when the vector was oblique, François placed the ruler on the vector and made it slide by conserving its orientation. However, this rudimentary procedure is inaccurate since the square is not used and the ruler moved when manipulated.

Although the logical-physical comprehension remained inaccurate, some elements seemed important to mention. First, to evaluate the logical-physical abstraction and logical-physical procedural comprehension, one of the tasks requested in the interview was to perform the translation of a cylinder and an octagonal prism on the face of which a triangle is represented in order to observe the conservation of the orientation of the solid. Regardless of the solid, rather then doing a parallel slide as expected, François raised the object and replaced it at the point of arrival without changing the orientation. When questioned about the procedure he chose, François answered: "Because, if I had it…well it has to arrive at the same place [according to the orientation]. If I had made it roll, it probably would not have arrived at the right place." In short, it was the final result of the translation that counted. This aspect also appeared at the level of intuitive comprehension when François should have identified concrete situations representing a translation. He then explained that a big wheel can represent a translation if the seats return to their starting position when it stops turning.

Concerning the analysis of mental strategies and utilization procedures of *Cabri Geometry*, let us point out that our analyses allowed us to observe, after Hölz (1996) who analyzed the effects of the drag mode on the learning of geometry, two types of activities: perception and comparison of examples and counter-examples. These activities were either static or dynamic. The first one dealt with the perception or comparison that is made when observing only examples or counter-examples on sheets of paper or on the computer screen. The second corresponds to the observation that was produced by the dynamic manipulation of examples or counter-examples on the computer. As a result of these observations, we were able to identify a particularity. The particularity concerned the manipulation of François with the *Cabri Geometry* software in that he mainly focused on examples during activities of perception and comparison. In fact, he attached little importance to counter-examples by justifying that it was in the examples that he was able to discover the essential characteristics of translation. Subsequently, during the same activities, François functioned in a more dynamic fashion. It was discovered that because of the drag mode, he spent a lot of time moving the diverse elements presented on the screen, the vector in particular.

Learning Acquired by Jean

Similar to François, Jean made considerable progress in the area of logical-mathematical comprehension of the concept of translation. Regarding formalization, Jean had a good idea of the role of the vector. His comprehension however appeared to be less clear than François, because he mentioned that the vector served to indicate where the figure should go, without naming specifically that the vector serves to indicate distance and sense. Jean's comprehension seemed to have evolved more in the area of logical-mathematical abstraction. In fact, he easily grasped that a figure conserves, during translation, its orientation, measures of length and angle. Moreover, we questioned him on his capacity to find the measurement of one side of the transformed figure for which the measurement of the corresponding side on the initial figure was indicated. He answered: "Even though it's the same measure as the initial figure because it's a translation." Concerning the procedural logical-mathematical comprehension, the results showed that Jean was capable of doing a translation on paper by respecting the information given by the vector, and by assuring himself that the transformed figure was identical to the initial figure.

Concerning logical-physical comprehension, Jean was able to perform a translation similar to that of François. He raised the solid piece and replaced it at the point of arrival. This method illustrated an inaccurate logical-physical procedural comprehension. The comprehension of Jean associated with the logical-physical abstraction seemed similar to that of François, for just like him he seemed to be concerned only with the final result of the translation and not with what occurred when the object was moved. He explained that, preferably, this movement is not done by rolling, for in such a case it is uncertain that the object conserves the same orientation as at the beginning. On the contrary, Jean believed that rolling may be associated with the translation, if the object is brought back to its initial orientation.

According to our interpretation of the explanations given by Jean with regard to a parallel slide, we may say that his intuitive comprehension remains inaccurate. As a matter of fact, for Jean, an object must slide easily, meaning to say without friction in order to be able to turn. Hence, according to him, an object that turns is one that slides at the outset, and, consequently this object effects a parallel slide - that is considered a translation. As for the procedures and strategies that Jean used during the teaching-learning sequence, we noted that, contrary to François, Jean barely used the dynamisms offered by *Cabri Geometry* for his activities of perception and comparison were mainly performed in a static fashion. Moreover, during this first phase of the sequence, Jean observed and compared as many examples as counter-examples.

Discussion

Like Gomes and Vergnaud (2004), who also analyzed conceptualization during the use of a DGS, it is supposed that the differentiated utilization of *Cabri Geometry* by the two pupils in our study, one exploiting mostly his dynamic potential than the other, explains their different comprehension of the concept of translation. In fact, the numerous dynamic manipulations made by François with the drag mode, in particular those associated with the vector, enable him to verify the rule of the vector in translation. As for Jean, his attention was focused more on the observation of the properties of the figures during translation. His activities of perception and comparison were mainly static and therefore did not exploit the possibilities offered by the *Cabri* software. These results correspond with those of the works of Rabardel (2000) and Trouche (2005) who showed that tools are not neutral, but rather they constituted a form of instrumental mediation since they influence the process of conceptualization. During the utilization of a given tool for learning, during the course of what Rabardel (2000) called ‘instrumental genesis’, pupils develop different instruments and different means of working with the same tool. The utilization of a tool is thus personalized by pupils who make the tool their own instrument of work.

Moreover, likely due to the technological environment, logical-physical comprehension remained inaccurate for the two pupils with regard to the translation. At the beginning of the study, we assumed that the implementation of a dynamic environment, such as the one available with the *Cabri Geometry* software, would allow us to notice a greater emergence of the movement associated with translation. In this way, the results suggested that the possibilities offered by *Cabri* could have misled the pupils. Effectively, François, at a given time, displaced the vector very quickly by making it turn. Since the software automatically actualizes the data of the translation, François was able to associate the movement of the translation with that of the rotation. Moreover, during the final phase of the teaching-learning sequence, the pupils themselves had to do a translation with *Cabri*. The manner in which they chose to do so could have led them to focus on the final result. As a matter of fact, a translation with *Cabri Geometry* requires the selection of an initial figure and clicking on the vector. In this way, the transformed figure ‘appears’ at the right place. If the pupil does not manipulate the vector afterwards, he or she is not aware of the type of movement made during the translation. Therefore, from an educational point of view, the didactic use of a tool should ideally rely on taking into consideration its constraints and potentialities. This equally suggests anticipation and management of the instrumental genesis (Rabardel, 2000). In other words, it is up to the teacher to plan and manage the different possible ways of utilization for pupils in relation to the constraints and possibilities of the tool.

Conclusion

From a practical point of view, one of the most interesting results to be considered is the capacity that the two pupils had to perform translations on paper with traditional geometric instruments, while working in a computer technology environment. Hence, teachers in school environments may gather from this study that technical learning, such as the handling of geometric instruments, are secondary compared to comprehension of a cognitive order. When pupils understand the properties of translation, they seem to find ways to proceed in order to respect these properties. The results also questioned the choice of a computer technology learning environment in mathematics and the role of teachers in such an environment. As stated by Sutherland and Balacheff (1999) different ‘micro-worlds’ contribute toward the construction of different meanings. In this sense, the use of *Cabri Geometry*, or any other computerized tool, must be used complementarily with a traditional environment (i.e. paper-pencil method). As stated in the study, the deployment of *Cabri Geometry* did not bring about work on the logical-physical plane. Moreover, it seems necessary that learning be supervised by the teacher for, according to Sinclair as well as Healy and Hoyles (2001), the strategies linked to the drag mode are not, as a rule, used by pupils. Analysis of teaching practices that implement computerized environments in mathematics is an area that deserves further attention and research.

References

Assude, T. & Gelis, J.-M. (2002). La dialectique ancien-nouveau dans l'intégration de Cabri-géomètre à l'école primaire. *Educational Studies in Mathematics*, *50*, 259-287.

Bardin, L. (2003). *L'analyse de contenu* (1re édition 1977). Paris: Presses Universitaires de France.

Barth, B.-M. (2001). *L'apprentissage de l'abstraction* (1re édition 1987). Paris: Éditions Retz.

Battista, M. T. (2001). A research-based perspective on teaching school geometry. In J. Brophy (Ed.), *Subject-specifici instructional methods and activities*: *Advances in research on teaching,* 8 (pp.145-185). Amsterdam: Jai Press/Elsevier Science.

Corriveau, A. & Morelli, C. (2005). Les transformations géométriques à l'ère des TIC. *Vivre le primaire*, *19*(1), 35-37.

Corriveau, A. (2007). *Étude de l'utilisation d'un logiciel de géométrie dynamique comme soutien au processus de conceptualisation et de compréhension du concept de translation chez des élèves du troisième cycle du primaire*. Mémoire de maîtrise non publié, Université de Sherbrooke, Québec.

Gomes, A. & Vergnaud, G. (2004). On the learning of geometric concepts using dynamic geometry software. *Novas tecnologias na educação*, *2*(1), 1-20.

Gouvernement du Québec. (2001). *Programme de formation de l'école québécoise*. *Éducation préscolaire*. *Enseignement primaire*. Québec: ministère de l'Éducation, No 01-00374.

Healy, L. et Hoyles, C. (2001). Software tools geometrical problem solving: Potentials and pitfalls. *International Journal of Computers for Mathematical Learning*, *6*, 235-256.

Herscovics, N. & Bergeron, J. C. (1988). An extended model of understanding. In M. J. Behr, C. B. Lacampagne & M. M. Wheeler (Eds.), *Actes de la 10e rencontre annuelle du North American Chapter of the International Group of the Psychology of Mathematics Education* (pp. 15-22). Dekalb, Il: PME.

Hölzl, R. (1996). How does "dragging" affect the learning of geometry? *International Journal of Computers for Mathematics Learning*, *1*(2), 169-187.

Laborde, C. & Capponi, B. (1994). Cabri-géomètre constituant d'un milieu pour l'apprentissage de la notion de figure géométrique. *Recherches en didactique des mathématiques*, *14*(1-2), 165-210.

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics*, *44*(1-2), 151-161.

Nantais, N. (1992). *La mini-entrevue*: *Un nouvel outil d'évaluation de la compréhension mathématique au primaire*. Thèse de doctorat non publiée, Université de Montréal, Montréal.

Organisation de Coopération et de Développement Économiques (OCDE). (2001). *L'école de demain*. *Les nouvelles technologies à l'école*: *apprendre à changer*. Paris: Centre pour la recherche et l'innovation dans l'enseignement (CERI).

Perrin-Glorian, M.-J. (2003). Studying geometric figures at primary school from surfaces to points. In M. A. Mariotti (Ed.), *Actes de la 3e conférence de l'European Society for Research in Mathematics Education* (pp. 1-10). Bellaria, It: CERME.

Rabardel, P. (2000). Éléments pour une approche instrumentale en didactique des mathématiques. In M. Bailleul (dir.), *Les instruments dans la pratique et l'enseignement des mathématiques.* Actes de l'école d'été de didactique des mathématiques (pp. 203-213). Caen, Fr: IUFM de Caen.

Sinclair, M. P. (2003). Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials. *Educational Studies in Mathematics*, *52*, 289-317.

Sutherland, R. & Balacheff, N. (1999). Didactical complexity of computational environments for the learning of mathematics. *International Journal of Computers for Mathematical Learning*, *4*, 1-26.

Trouche, L. (2005). Construction et conduite des instruments dans les apprentissages mathématiques: nécessité des orchestrations. *Recherches en Didactique des Mathématiques*, *25*(1), 91-138.

Notes

[1] The conceptualization can not always rely on the presentation of examples and counter-examples. Through this research, we made this choice because the theoretical approach proposed by Barth (2001) is well suited to learning the translation.

[2] Both students come from a school in the Appalachian School Board.

[3] In this research, we do not distinguish the direction of the vector from its sense because, although both terms are used by students, they are employed in undifferentiated ways.